proposition takes the following form: *The diameter passing through a variable point of a parabola, meets the tangent at the vertex in the point P. The parallel through P to the line joining M to the vertex of the parabola, envelops another parabola having the same vertex and the same axis as the given curve.*

The proposed problem is another special case of this general proposition, namely when both A and B are at infinity.

The duals of the three propositions are, in order:

The points of intersection of two fixed tangents to a given conic, with a variable tangent to the same curve, are projected from the points where the fixed tangents touch the conic. The point of intersection of the two projecting lines describes a conic having a double contact with the given curve.

From a variable point of the tangent at the vertex of a given parabola, are drawn the diameter and the tangent to the curve. The point of intersection of the diameter with the parallel to the tangent through the vertex of the curve, describes a parabola having the same axis and the same vertex as the given curve.

The parallels to the asymptotes of a given hyperbola drawn through the points of intersection of the latter lines with a variable tangent to the curve, intersect in a point whose locus is an hyperbola having the same asymptotes as the given curve.

Also solved by O. S. Adams, Clara L. Bacon, J. W. Clawson, A. M. Harding, Horace Olson, Paul Capron, G. W. Hartwell, R. M. Mathews, and N. P. Pandya.

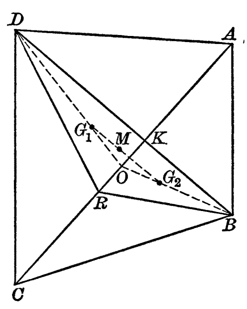
490. Proposed by ELMER E. MOOTS, University of Arizona.

In any quadrilateral $ABCD$, let AC and BD be the diagonals intersecting at K. On AC, lay off CR equal to AK. Join B and R. Connect the middle point G of BR with D. On GD lay off GM equal to $\frac{1}{3}GD$. Show that M is the center of gravity of the quadrilateral.

Solution by A. M. Harding, University of Arkansas.

It is evident that M is the center of gravity of the triangle BDR. Hence it will be sufficient to prove that the triangle BDR and the quadrilateral $ABCD$ have the same center of gravity.

Let O be the mid-point of RK, then it will also be the mid-point of CA. Then G_1 is the center of gravity of the triangles RDK and CDA, and G_2 is the center of gravity of the triangles RBK and CBA where $OG_1 = \frac{1}{3}OD$ and $OG_2 = \frac{1}{3}OB$.



Since

$$\frac{\triangle RDK}{\triangle RBK} = \frac{\triangle CDA}{\triangle CBA},$$

is follows that the center of gravity of the triangle BDR will also be the center of gravity of the quadrilateral $ABCD$.

Also solved by J. W. Clawson, O. S. Adams, and N. P. Pandya.

491. Proposed by N. P. PANDYA, Sojitra, India.

In a triangle $mx = b$ and $nx = c$, determine a relation between m, n, x, A and s, and solve it for x.

SOLUTION BY J. A. COLSON, Searsport, Maine.

Since $b = mx$ and $c = nx$, we have $\sin^2 A/2 = (s - b)(s - c)/bc = (s - mx)(s - nx)/mnx^2$ Hence, $mnx^2 \sin^2 A/2 = s^2 - (m + n)sx + mnx^2$, or $mnx^2 \cos^2 A/2 - (m + n)sx + s^2 = 0$.

Solving this quadratic for x, we have

$$x = \frac{(m + n)s \pm s\sqrt{\{(m + n)^2 - 4mn \cos^2 A/2\}}}{2mn \cos^2 A/2}.$$

492. Proposed by FRANK V. MORLEY, Student, Haverford College.

Let a_i $(i = 1, 2, 3, 4)$ be four points on a circle, and let the symmedian point of the triangle formed by omitting a_i be s_i. Prove that the four points s_i have the same diagonal triangle as the four points a_i.

SOLUTION BY J. E. ROWE, Pennsylvania State College.

We choose that system of homogeneous coördinates in which the coördinates of a point are proportional to $\alpha/a : \beta/b : \gamma/c$, where a, b, c are the lengths of the sides of the reference triangle and α, β, γ the lengths of the $\perp$'s from the sides to the point. It may easily be shown that in this system of coördinates the equation of the circle circumscribing the reference triangle is

$$(1) \qquad x_2x_3 + x_1x_3 + x_1x_2 = 0.$$

Let the coördinates of the four a's be

$$a_1 \equiv b_1, b_2, b_3; \quad a_2 \equiv 1, 0, 0; \quad a_3 \equiv 0, 1, 0; \quad a_4 \equiv 0, 0, 1.$$

There is evidently no loss of generality in this selection of the a's, and they will all lie on (1) if only

$$(2) \qquad b_2b_3 + b_1b_3 + b_1b_2 = 0.$$

The coördinates of P_1 the intersection of the lines a_1a_3 and a_2a_4 are $b_1, 0, b_3$; similarly the coördinates of P_2 the intersection of the lines a_1a_4 and a_2a_3 are $b_1, b_2, 0$; and the coördinates of P_3 the intersection of the lines a_1a_2 and a_3a_4 are $0, b_2, b_3$. That is, $P_1P_2P_3$ is the diagonal triangle of the a's.

The equations of the tangents to (1) at the points a_i are

$$\begin{aligned}
T_1 &\equiv (b_2 + b_3)x_1 + (b_1 + b_3)x_2 + (b_1 + b_2)x_3 = 0,\\
T_2 &\equiv x_2 + x_3 = 0,\\
T_3 &\equiv x_1 + x_3 = 0,\\
T_4 &\equiv x_1 + x_2 = 0.
\end{aligned}$$

The tangents to (1) at two of the points a intersect in a point, and this point and a third a determine a line. Any set of three a's yields three such lines which are concurrent through the symmedian point of the three a's. In this way we find that the coördinates of the symmedian points are

$$s_1 \equiv 1, 1, 1; \; s_2 \equiv b_1, b_1 + 2b_2, b_1 + 2b_3; \; s_3 \equiv b_2 + 2b_1, b_2, b_2 + 2b_3; \; s_4 \equiv b_3 + 2b_1, b_3 + 2b_2, b_3.$$

By reason of (2) the determinant

$$\begin{vmatrix} b_1 & b_1 + 2b_2 & b_1 + 2b_3 \\ b_3 + 2b_1 & b_3 + 2b_2 & b_3 \\ b_1 & 0 & b_3 \end{vmatrix} = 0.$$

Hence, the points s_2, s_4, and P_1 are collinear. In the same way it may be shown that s_1, s_3, and P_1 are collinear. From the symmetry of the coördinates of the P's and the s_i it follows that $s_2s_3P_2$, $s_1s_4P_2$, $s_1s_2P_3$, and $s_3s_4P_3$ are collinear sets of three points, and this shows that $P_1P_2P_3$ is the diagonal triangle of the s_i.

Also solved by J. W. CLAWSON and J. W. HASLEY.